Probability Model for Cumulative Solar Proton Event Fluences

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Abstract

A new model of cumulative solar proton event fluences is presented. It allows the expected total fluence to be calculated for a given confidence level and for time periods corresponding to space missions. The new model is in reasonable agreement with the JPL91 model over their common proton energy range of > 1 to > 60 MeV. The current model extends this energy range to > 300 MeV. It also incorporates more recent data which tends to make predicted fluences slightly higher than JPL91.

For the first time, an analytic solution is obtained for this problem of accumulated fluence over a mission. Several techniques are used, including Maximum Entropy, to show the solution is well represented as a lognormal probability distribution of the total fluence. The advantages are that it is relatively easy to work with and to update as more solar proton event data become available.

I. INTRODUCTION

The effects that solar proton events have on microelectronics and solar arrays are a significant concern for spacecraft in geostationary and polar orbits, and on interplanetary missions. In planning for space missions it is useful to have information about the worst case event that will be encountered and about the cumulative solar proton event fluence over the entire mission. Two models commonly used for cumulative fluence estimates are the SOLPRO model [1], based on King's analysis of spacecraft measurements of solar cycle 20 data [2], and a model from JPL which was initially based on ground data from solar cycle 19 and spacecraft measurements from cycles 20 and 21 [3]. The updated JPL91 model incorporates cycles 20, 21 and part of 22 [4]. The SOLPRO model was based on a solar cycle that had one extremely large event that dominated the total fluence of that cycle (the August 1972 event). The model therefore predicts the number of such extremely large events expected for a

given mission length and confidence level. Using additional data from solar cycles 19 and 21, the Feynman team later showed that the severity of solar proton events actually forms a continuum between small events and the "anomalously large" event of August 1972.

The SOLPRO and JPL models have been very useful for predicting event fluences for long-term degradation but do have limitations due to the incomplete nature of the data sets upon which they were based. The first limitation is the proton energy range. The SOLPRO model covers the energy range > 10 to > 100 MeV and the JPL91 model covers energies > 1 to > 60 MeV. Fluence levels below 10 MeV are desirable for accurate predictions of solar cell degradations, whereas, the higher energy protons, with their ability to penetrate shielding, are important to consider for total dose degradation and single event effects in system electronics. Clearly, a model that has adequate energy range for all applications is needed. The second limitation is that neither previous model includes the full 3 solar cycles for which high quality space data are available. This is important because the 3 cycles were dissimilar from one another. Cycle 20 had one anomalously large event that accounted for most of the accumulated fluence. Cycle 21 was a rather quiet cycle with no such large events. Cycle 22 was very active and had several very large events.

Another reason for reassessing the cumulative solar proton event fluence models at this point is that a new and accurate approach has recently emerged for describing the underlying or initial distribution of solar proton event fluences [5]. It is based on Maximum Entropy Theory [6], and predicts an initial distribution that is a truncated power law in the event fluence. An example of this is shown in Figure 1, which is a plot of the number of events per solar active year that exceed a given event fluence vs. fluence. The points represent the measured > 30 MeV event fluences during active years of solar cycles 20-22. These are compared to the distribution predicted by the Maximum Entropy technique, shown by the line. This approach is a significant improvement in describing the distribution of events compared to previous empirical

methods such as those using lognormal distributions [2-4] and power laws [7]. Since a model of cumulative fluence must be based on some initial distribution of event fluences, it is worthwhile to use the improved distribution obtained from the Maximum Entropy approach.
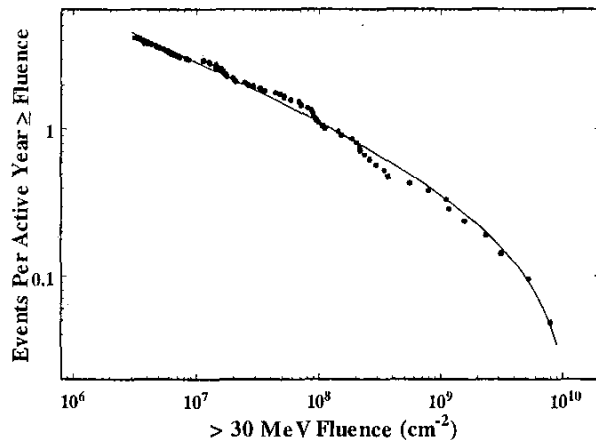

Figure 1. Distribution of > 30 MeV solar proton event fluences. The points are satellite data from active years of solar cycles 20-22, and the line is a truncated power law distribution based on Maximum Entropy Theory.

In section II, the solar proton event data used in this study are described. Section III gives an overview of the methods. First, it is shown that an analytic expression can be obtained for the cumulative fluence distribution. Next, it is shown how the distribution parameters are obtained for any given confidence level and period of time. This statistical model is applicable to proton energies from > 1 to > 100 MeV. It is then shown that solar cycle 22 data can be used to extrapolate these results out to > 300 MeV. Section IV presents results and compares them to the JPL91 predictions. Finally, section V gives conclusions.

II. SOLAR PROTON EVENT DATA

Solar proton event data from the last 3 complete solar cycles (20-22) were processed at NASA-Goddard Space Flight Center to obtain the event fluences. The source of the data for cycle 20 was the IMP-3, -4, -5, -7 and -8 satellites. The data from cycle 21 was from IMP-8. The GOES-5, -6 and -7 satellite data, which extends to much higher proton energies, was used for cycle 22. The particular detectors and other flight details are discussed elsewhere [8]. The fluence data obtained are similar to that published by King for cycle 20 [2], Goswami for cycle 21 [9] and Stassinopoulos for cycle 22 [10]. The previous data analyses of Feynman [3] and Shea and Smart [11] were also reviewed for this work.

Only events having some minimum fluence were considered. This was dependent on proton energy as tabulated in reference 5. The minimum value was very much smaller (several orders of magnitude) than the cumulative fluences predicted by the model. Thus, the choices of lower limits do not affect our results. In identifying events, we have followed the practice of NOAA, as published in *Solar Geophysical Data Reports* [12], where the beginning and end of an event are identified by a threshold proton flux so that a large event may consist of several successive rises and falls in flux.

Since a very large portion of the accumulated fluence occurs during solar active years, it is reasonable to neglect the solar inactive years [3,4]. Our definition of a solar active year is based on the work of Feynman [4]. A solar cycle typically lasts 11 years - 7 active and 4 inactive. The 7 active years are assumed to span a starting point 2.5 years before and an ending point 4.5 years after a time defined by the maximum sunspot number in the cycle. These times for the last 3 solar cycles were 1968.9, 1979.9 and 1989.9.

III. METHODS

A. Form of the Cumulative Fluence Distribution

The Maximum Entropy Principle has been successfully used to select the initial distribution of solar proton event fluences [5] and peak fluxes [13]. It provides a mathematical procedure for selecting a probability distribution when the data are incomplete. It has been argued that this is the best choice that can be made given limited data [6,14]. Here, we use it to guide our selection of a distribution for cumulative fluences. The general procedure for application to solar proton events has been described previously [5,13] and will not be repeated here. The following 4 constraints are applied to the distribution in determining its form:

1. It can be normalized.
2. It has a well defined mean value. This follows from knowing the form of the initial distribution.
3. It has a well defined variance. This also follows from knowing the form of the initial distribution.
4. Only positive values of cumulative fluences are allowed, and they are unbounded in the positive direction. This follows from the Poissonian nature of the number of events [4].

Under these conditions the Maximum Entropy Principle shows that the best choice of a probability distribution for the cumulative fluence, Φ, is a lognormal distribution. An analogous although purely mathematical description of arriving at the choice of a lognormal distribution is given in Kapur [6].

We have also performed a variety of simulations to validate this choice of distribution. For example, summing simulated event fluences determined by an initial distribution such as that shown in Figure 1, and assuming the event numbers are Poisson probabilities results in a lognormal distribution in the cumulative fluence. Bootstrap-like methods

also indicate that a lognormal distribution is appropriate, and holds for periods of time up to at least 7 solar active years [15]. Probably the most direct method, however, is to examine actual satellite data for a given period of time. This is shown in Figure 2 for 1 year intervals during solar active periods. The y-axis shows the summed fluence of events during each solar active year. The cumulative probability of each 1 active year fluence total is calculated as $m/(N+1)$, where m is the rank and N is the number of data points [16]. The probability paper used for Figure 2 is constructed so that a lognormal distribution appears as a straight line. Thus, it is seen that the cumulative fluence distributions are well described as lognormal. In order to interpret this figure, note that for a cumulative probability (equivalent to confidence level) of 0.90, the annual fluence for >100 MeV protons is about $2.6 \times 10^8 cm^{-2}$. This means that 90% of the fluences for 1 active year periods are less than or equal to $2.6 \times 10^8 cm^{-2}$.
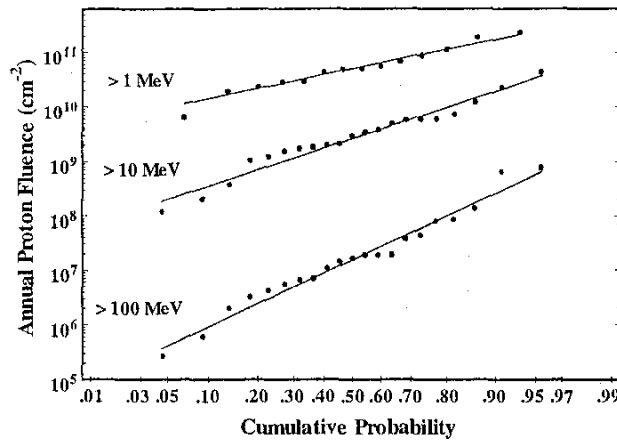

Figure 2. Probability plot, constructed on lognormal probability paper, for the total annual solar proton event fluence observed at 1 AU during solar active years. Satellite data for solar cycles 20-22 are shown for proton energies > 1, > 10 and > 100 MeV.

Thus, the distribution of cumulative fluences, Φ, for any period of time can be written as the cumulative lognormal function

$$F_{CUM} = \frac{1}{\sigma\sqrt{2\pi}} \int_{0^+}^{\Phi} \frac{1}{\Phi'} \exp\left\{-\frac{1}{2\sigma^2}\left[\ln(\Phi') - \mu\right]^2\right\} d\Phi' \tag{1}$$

The value of F_{CUM} is the confidence level for observing a total proton fluence Φ over a time period of T active years. The lognormal parameters σ and μ are dependent on T in a simple way. They can be obtained from the lognormal parameters of the fitted total annual fluence distributions such as those shown in Figure 2. This is described next. It should be noted that this approach is distinctly different from the JPL91 model, which is a Monte Carlo based approach.

B. Calculation of the Lognormal Distribution Parameters

The total fluence over the course of a space mission is specified by equation (1). The parameters σ and μ for a time period of T solar active years can be obtained from the distribution for $T = 1$ solar active year as follows. The lognormal parameters for 1 active year are taken as the best fit values obtained from probability plots such as those shown in Figure 2. These parameters are related to the mean fluence and its relative variance of the 1 active year distribution by the following [17]:

$$\Phi_{mean} = \exp\left(\mu + \frac{\sigma^2}{2}\right) \tag{2}$$

$$\Phi_{RV} = \exp(\sigma^2) - 1 \tag{3}$$

The mean fluence and its relative variance for a T active year distribution follow from compound Poisson process theory [18]. (A compound Poisson process is one where the number of events follows Poisson statistics and the magnitude of events is a random variable, as is the case with solar proton events.) The mean is $T \times \Phi_{mean}$ and the relative variance is Φ_{RV}/T. The relations shown by equations (2) and (3) can now be used to calculate the lognormal parameters for a T active year distribution.

Inserting the calculated lognormal parameters for a T active year distribution into equation (1) allows the cumulative fluence to be obtained for any desired confidence level. This equation must be evaluated numerically. However, this is easily accomplished because a variety of software and tabulated values of normal and lognormal functions exist and can be used to evaluate such an equation. These calculations have been done for proton energies ranging from > 1 to > 100 MeV.

C. Extrapolation to Higher Energies

The solar proton event data at energies higher than 100 MeV is rather limited. This prevents the implementation of the probabilistic model at these energies. However, the available data from GOES satellite measurements during solar cycle 22 were examined for their spectral shape. This included the large events of 8/12/89, 9/29/89, 10/19/89, 3/23/91, 6/4/91 and 10/30/92, their summed spectra, and the CREME96 [19] worst day spectrum. Comparisons were made to the statistical model for > 1 to > 100 MeV protons, confidence levels between 0.50 and 0.99, and mission lengths ranging from 1 to 7 solar active years. Based on these comparisons it was determined that the spectral shape for these GOES measurements between > 100 and > 300 MeV is a good approximation for extending the probabilistic model spectra. Thus, the > 100 to > 300 MeV GOES data are scaled

to extrapolate the probabilistic model spectra out to > 300 MeV.

IV. RESULTS

For many radiation effects applications it is desirable to know the fluence-energy spectrum of the cumulative solar proton event distributions. Typical examples of this are shown in Figure 3 for a time period of 1 solar active year. The ordinate represents the cumulative solar proton event fluence at threshold energies ranging from > 1 to > 300 MeV. Spectra are shown for confidence levels of 0.50, 0.80, 0.90 and 0.99. The points represent results obtained using the methods described in section III. The statistical model results cover the energy range between > 1 and > 100 MeV, and are extrapolated out to > 300 MeV. Thus, for example, there is a 50% chance that the > 30 MeV proton fluence will not exceed 3.7×10^8 cm^{-2} during 1 active year. There is an 80% chance it will not exceed 1.7×10^9 cm^{-2}, a 90% chance it will not exceed 4.0×10^9 cm^{-2}, and a 99% chance it will not exceed 2.8×10^{10} cm^{-2}.
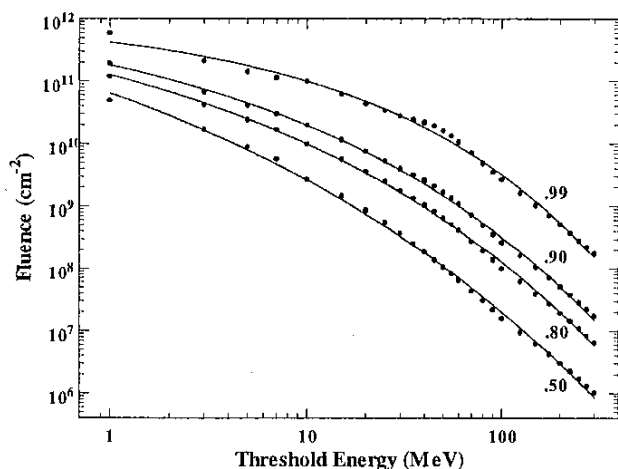

Figure 3. Fluence-energy spectra for a time period of 1 solar active year, and for confidence levels of 0.50, 0.80, 0.90 and 0.99. The model calculations are shown by the points. The lines are an empirical fit. See text.

A physically-based form of the fluence-energy spectra is not currently known. However, we have determined an empirical approximation that is given by

$$\Phi = \Phi_0 \exp(-kE_{th}^a) \qquad (4)$$

where E_{th} is the threshold energy in MeV, and Φ_0, k and a are fitted constants. The fits are shown by the lines in Figure 3. It is seen that they describe the energy spectra reasonably well over the full range of data. The fitted constants generally depend on confidence level and mission duration. For the

90% confidence level the values are $\Phi_0 = 2.40 \times 10^{12}$, $k = 2.59$ and $a = 0.269$.

Results have also been obtained for longer duration missions and are shown in Figure 4 by the lines. Here the y-axis represents the accumulated fluence for mission lengths between 1 and 7 solar active years. This is shown for proton energy thresholds of > 1, > 10, > 30, > 60, > 100, > 200 and > 300 MeV. All results in Figure 4 are for the commonly used 90% confidence level. These results are compared to the JPL91 model predictions [4], shown by the points. It is seen that the two models are in reasonable agreement. This is especially encouraging when one considers how different the two model approaches are. A minor difference in the results is that the current model (ESP) tends to give slightly higher fluences than JPL91, which is more pronounced at lower energies. Aside from the different approaches taken, there are two other factors that may contribute to this. The first is that the ESP model incorporates the last 3 full solar cycles (20-22), whereas JPL91 incorporates cycles 20, 21 and part of 22. Since cycle 22 was rather active, this difference in the two data bases will tend to make the ESP fluence predictions higher than JPL91. The second factor, although likely a less significant one, is that JPL91 is based on an *initial* distribution that is lognormal, which underestimates the number of small solar proton events [4]. ESP is based on an initial distribution that is a truncated power law, and accurately describes the complete initial distribution, as shown in Figure 1.
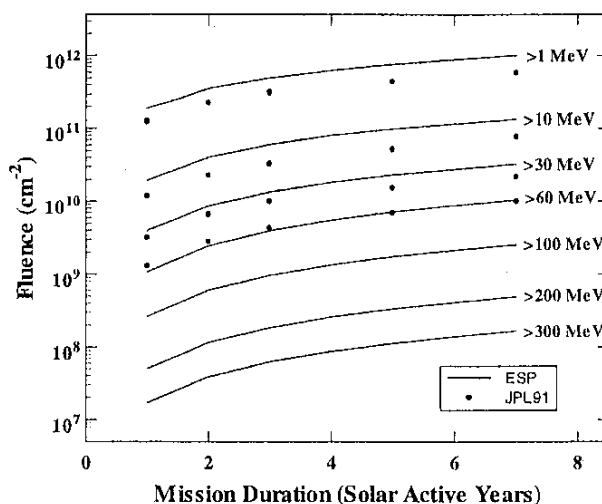

Figure 4. Predictions of accumulated fluence at the 90% confidence level for mission lengths ranging from 1 to 7 solar active years. Energies between > 1 to > 300 MeV are indicated. Comparisons of the current ESP model (lines) are made to the JPL91 model (points) over their common energy range of > 1 to > 60 MeV.

V. SUMMARY AND CONCLUSIONS

We have presented a new model for predicting cumulative solar proton event fluences for any confidence level and for time periods corresponding to realistic space missions. The model is based on a more complete data set than previous models. It incorporates data from the last 3 complete solar cycles, and includes proton energies ranging from > 1 to > 300 MeV. When compared to the JPL91 model over its range of applicability reasonable agreement is seen considering the different approaches and data bases.

Several statistical methods have been used to demonstrate that the cumulative fluence distribution is well described as one that is lognormal in fluence. This analytic result is advantageous because it is simpler to work with and to update compared to Monte Carlo based approaches. Furthermore, it has been formulated concurrently with a probabilistic model of worst case events [5], thus providing a more complete complement of tools for space applications. Both models are available in a Windows compatible program [8].

ACKNOWLEDGMENT

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